# A Game Theory Model of Product Sampling in New Product 

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#### Abstract

This paper discusses the use of product sampling as a marketing tool to reduce consumers' uncertainty and help them learn about new products before making a purchase decision. The paper provides examples of different product sampling strategies used for various product types and introduces a game theoretic model to find the optimal level of product sampling for companies. The paper assumes that consumers have a prior belief about the new product and that their impression of the product is formed during the product sampling. The study shows that more informative product sampling can increase the likelihood of a consumer with a high prior belief having a favourable impression and a consumer with a low prior belief having an unfavourable impression. This paper summarises how companies in different industries should determine the optimal product sampling level to maximise profit.


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## 1. Introduction

When a new product is released, consumers are uncertain whether or not it will meet their needs well. To reduce this uncertainty, companies can utilise product sampling to provide consumers with an opportunity to learn about their preference over this new product before finalising their purchase decision.

It is evidenced that product sampling is one of the most effective and widely adopted tools of marketing in many industries. For example, the food industry Frito-lay distributes more than 6 million packets of Corn and Cheese Doritos in a single day and spends more than \$3 million to launch the new snack product (Brandweek, 1995). The digital product industry also offers free trials to customers. These samples can be specific (e.g., a free download of a particular song) or general (e.g., a $\$ 5$ voucher from an online music store) (Kasper, 1996). LinkedIn provides product sampling with limited functions to the users to explore their premium version. As shown in Table 1, users can select and buy their preferred version after their free-trial experiences. Nevertheless, Microsoft provides product sampling with limited time but full functionality.

| Premium subscription plan | Premium <br> Career | Premium <br> Business | Sales <br> Navigator <br> Core | Recruiter Lite | Linkedln <br> Learning |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Private Browsing | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |
| Applicant Insights | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |
| Direct messaging | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |
| Who Viewed Your Profile | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |
| Unlimited access to LinkedIn Learning | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Unlimited people browsing |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |
| Business Insights |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |

Figure 1. Linkedin Premium Subscription Plan

There is no doubt that the specific format of product sampling strategy depends on the industry and the nature of products. The literature has documented massive product sampling strategies. Based on a comprehensive review, we summarise eight product sampling strategies and their corresponding industry in Figure 2. Despite the distinct differences across various product sampling strategies, for each product sampling strategy, there is a question of the extent of product sampling level. In other words, how far should a product sampling strategy go? For example, in the traditional sampling, how many free samples should be given for each consumer? What is the optimal time length in a limited time sampling strategy? Figure 3 shows different product sampling strategies and their corresponding levels. Companies in different industries can choose which and how many product sampling strategies to use to convey the information about the new product to consumers, which is a choice variable for the companies.

| Product Sampling Strategy |  | Context |
| :---: | :---: | :---: |
| Traditional Sampling | Give away free product samples to customers | Medical Health; Cosmetics; Food |
| Cashback sampling | Coupon, or receive money back after uploading the receipt | Apparel; Medical Health; Software; Cosmetics; Food |
| Sampler packs | Small or trial-size products | Medical Health; Cosmetics; Food |
| Mail drops | Mail free samples | Medical Health; Cosmetics; Food |
| Limited time samples | Free only for a specific amount of time | Apparel; Medical Health; Software; Cosmetics |
| Limited function samples | Free only for some specific functions | Software |
| Virtual sampling | Consumers are able to try out a product online | Apparel; Cosmetics |
| Social content samples | Social networking sites, extract from the comments | Apparel; Medical Health; Software; Cosmetics; Food |

Figure 2. Different Product Sampling Strategies in Different Contexts

The literature has documented the effectiveness of a single product sampling strategy. For example, Lin et al. (2019) empirically analysed the effect of product participation in free product sampling on product evaluation score, and investigated the important contingent factors of product pricing and product popularity. Furthermore, other studies shed light on the comparison between different product sampling strategies. For instance, Cheng et al. (2012) study the context where software companies can choose to offer free trial software with full features but limited trial time, or free trial software with limited features but unlimited trial time. Nevertheless, our knowledge of the optimal level of product sampling is still limited. To fill this research gap, in our study, we develop a game theory model to find the optimal value
for this choice variable to help the company decide what is the optimal level of the product sampling.

| Product Sampling Strategy | Level |  |
| :---: | :---: | :---: |
| Cashback sampling | Return rate, amount of coupon | Coupon, or receive money back after uploading the |
| receipt |  |  |

Figure 3. Different product Sampling Strategies with Different Levels

There is a trade-off when a seller decides the product sampling level. On the one hand, increasing the product sampling level can increase high type user proportion, and the willingness to pay from the high-type consumer will also increase. This is called the Learning Effect; on the other hand, increasing the product sampling level will decrease the demand from low-type consumers and the willingness to pay from low-type consumers. This is called the Market Division Effect.

In our study, we conceptualise product sampling as a signal device since it fundamentally reduces consumers' uncertainty such that it signals consumers' true valuation over a new product. Mathematically, it is a mapping from the unobservable true product valuation (prior belief) to the observable signal realisation - product impression after sampling. For simplification, we assume that before the product sampling, there are two types of consumers: high type (mass $\theta$ ) who has a high valuation $\left(v_{H}\right)$ over the new product and low type who has a low product valuation $\left(v_{L}\right)$.A consumer may have a high prior belief and also may have a low one, which means the consumer may be interested in buying this new product or may be not interested in it and the new product unsuitable for this consumer.

During product sampling, consumers privately observe a signal realisation, which we call their "impression" of the new product. Consumers with an "unfavourable" impression believe that the new product does not meet their needs. Those with a favourable impression
believe that the new product is more likely to meet their needs, but they may not be entirely sure that this is the case. After the product sampling, a consumer will have a posterior belief with a "favourable" or an "unfavourable" impression. A more informative product sampling increases the likelihood that a consumer who is unsuitable for the new product will get an unfavourable impression. And on the other hand, a more informative product sampling increases the likelihood that a consumer who has a high prior belief for the new product will get a favourable impression. Moreover, consumers with a high prior belief and those with a low prior belief for this new product may react differently to the product sampling. This may be explained by the proportion of consumers with a low prior belief having an unfavourable impression after the product sampling which may be different from the proportion of consumers with a high prior belief having a favourable impression. (Boleslavsky, 2017).

The game theory model takes into account the different product sampling strategies used in various industries and their corresponding levels. It assumes that consumers have a prior belief about the new product and that their impression of the product is formed during the product sampling. By using game theory, our model provides a framework for understanding the interactions between companies and consumers, and helps companies make decisions that maximise their profits.

Overall, the purpose of our model is to provide a data-driven approach to decision-making for companies, which can improve the effectiveness of their marketing strategies and ultimately increase their revenue. The intuition behind the model is to help companies strike a balance between providing enough information to reduce consumers' uncertainty while minimising the cost of product sampling, which can be a significant expense for companies.

## 2. Literature Review

The prior work has documented the effectiveness of product sampling in physical goods (Bawa et al. 2004), digital experience goods (Chellappa and Shivendu 2005), the comparison between different product sampling strategies (Cheng et al. 2012). In this section, we briefly review these relevant studies and articulate how our work differs from them.

Bawa et al. (2004) used a game theory model to analyse the effects of product sampling and make the following conclusions: there are three potential effects of product sampling on sales: (1) an acceleration effect, whereby consumers begin repeat purchasing of the sampled produce earlier than they otherwise would; (2) a cannibalization effect, which reduces the number of paid trial purchases of the product; and (3) an expansion effect, which induces purchasing by consumers who would not consider buying the product without a free sample. While Bawa et al. (2004) analysed the effects of product sampling on sales using a game theory model, our study extends their work by introducing the notion of consumer uncertainty and how it influences the effectiveness of product sampling. In addition, we propose a more comprehensive model that considers the impact of different levels of product sampling on consumers with different prior beliefs, which can help companies determine the optimal level of product sampling to maximise their profits.

Chellappa and Shivendu (2005) develop a pricing model for digital experience goods in a segmented market, considering the effect of piracy on product evaluation and purchase decisions. The study finds that traditional segmentation and sampling recommendations need to be revisited for digital goods due to the presence of piracy. The authors recommend distinct pricing and sampling strategies for underestimated and overestimated products and suggest that piracy losses can be mitigated through product sampling. However, unlike physical goods, sampling for digital goods is only optimal under specific circumstances due to price boundaries created by piracy and segmentation. Piracy losses are more severe for products that do not live up to their hype, requiring greater deterrence investment. Chellappa and Shivendu's (2005) study is relevant to my research as it highlights the importance of piracy in the pricing and sampling strategies of digital experience goods. As digital piracy is still a prevalent issue in the software industry, I can incorporate their findings into my study by examining how different pricing and sampling strategies can be used to improve the overall profitability of software products.

Cheng et al. (2012) study the context where software companies can choose to offer free trial software with full features but limited trial time, or free trial software with limited features but unlimited trial time. In their study, they used a game theory model to study the tradeoff between the uncertainty reduction effect and demand cannibalization effect, and revealed the conditions under which software companies should introduce time-locked free trial software, and found the optimal free trial time. Based on this model, they further analysed the tradeoff between limited version software free trial strategy and time-lock software free trial strategy, which depends on network effects and the difference between the consumer's prior belief and the true functionality of the software. Based on Cheng et al.'s (2012) findings on the optimal free trial time and the tradeoff between limited version software free trial strategy and time-lock software free trial strategy, my study will examine the effectiveness of free trial duration on consumer behaviour in the context of a new software product.

Schlereth et al. (2014) propose an agent-based model to study the effectiveness of product-sampling campaigns based on social network information. The study finds that targeting the right consumers is more important than the number of consumers targeted. Using social network information can increase profits by at least $32 \%$, and a high-degree targeting heuristic should be used to identify influential consumers. Social network information is particularly effective for single-purchase products because it speeds up diffusion, while for repeat-purchase products, it reduces the optimal number of samples and the campaign cost.

Foubert et al. (2016) posit that product sampling is a double-edged sword. On the one hand, product sampling allows consumers to learn about the service for free, thereby generating new paying users. On the other hand, a disappointing trial experience may alienate potential customers when they decide not to adopt the system and are lost forever. In their study, they incorporated these phenomena into a model of consumer product sampling and regular adoption decisions, and used data from a large Western European telecommunications company that provides telephone, Internet, and television services and operates in a single country to do the simulation run. The results highlight that product sampling is a double-edged sword, with the length of the trial and the intensity of consumer use being key to the effectiveness of these promotions. Based on Foubert et al.'s (2016) findings that
product sampling can have both positive and negative effects on consumer adoption decisions, my study will also consider the potential drawbacks of product sampling.

Boleslavsky et al. (2017). develop a game theoretic model of price competition, incorporating product demonstrations as a means for consumers to learn about the value of a new product. Demonstrations may include product samples, trials, return policies, or online review platforms. The innovative firm controls the level of demonstration informativeness. When the innovative firm commits to demonstration policies and has price flexibility, fully informative demonstrations are preferred, dividing the market and reducing price competition. However, when a firm can adjust its demonstration strategy in response to prices, the firm prefers only partially informative demonstrations, maximising its market share and generating monopoly profits. The authors contrast the strategic role of demonstrations in their model with the strategic role of capacity limits in models of judo economics.Boleslavsky et al.'s (2017) study on the strategic role of product demonstrations is relevant to my paper because it provides insights on the optimal level of informativeness for demonstrations and how it affects market competition. This study shows that fully informative demonstrations can divide the market and reduce price competition, while partially informative demonstrations can maximise market share and generate monopoly profits. I can incorporate these findings into my paper when discussing the tradeoffs of different product sampling strategies and their impact on market outcomes.

Lin et al. (2019) empirically analysed the effect of product participation in free product sampling on product evaluation score, and investigated the important contingent factors of product pricing and product popularity. They found that participating in free product sampling improved product ratings by $1.1 \%$ using Taobao.com's dataset. It is the reciprocal behaviour of consumers giving higher ratings in return for beneficial behaviour toward retailers that causes the rating bias. When the original price is higher, the deviation is larger; When the price discount is larger and the product popularity is higher, the deviation is smaller.

Reza et al. (2020) systematically examined problems for current users both analytically and empirically. Their analytical discussion highlights why some current users may be effective targets for free sample promotions. On this basis, an empirical analysis is conducted using a free sample of pre-promotion and post-promotion mobile data from a telecommunications
company using the dataset. The experimental results are consistent with the analytical results. They find that the initial level of use is a key factor in determining the conversion rate of free samples and subsequent changes in use due to the conversion rate of free samples and discuss the implications of our findings for management and policy.

The literature review highlights several studies on the effectiveness of product sampling, pricing, and demonstrations in increasing sales and profits for companies. Bawa et al. (2004) showed that product sampling can have three potential effects on sales: an acceleration effect, a cannibalization effect, and an expansion effect. Chellappa and Shivendu (2005) found that digital piracy affects pricing and sampling strategies for digital goods. Cheng et al. (2012) explored the optimal duration of free trial software, while Schlereth et al. (2014) showed that targeting the right consumers is more important than the number of consumers targeted. Foubert et al. (2016) posited that product sampling can have both positive and negative effects on consumer adoption decisions. Finally, Boleslavsky et al. (2017) developed a game theoretic model that considers product demonstrations as a means for consumers to learn about the value of a new product, and how it affects market competition. The game theory models in past research only consider the impact of product sampling on the consumer with low prior belief. In this paper, I will extend the model including the impact for both low-type and high-type consumers, find the optimal product sampling level, with implications for firms to strategically decide the product sampling.

## 3. Model Setup

We model the interaction between a consumer and a monopolistic company in new product marketing. The seller produces a new product where neither the consumer nor the company know the consumer's true valuation of the new product. Nevertheless, the seller can adopt product sampling strategies and features. The production cost of the new product and the cost of the product sampling are normalised to zero, because these costs are not the focus of this research. The consumer is modelled as unit demand.

### 3.1. The Timing of Game

The sequence of the game is depicted in Figure 4.

| Stage 1: The monopolistic |
| :--- |
| company sets the product |
| sampling level s. |


| Stage 2: The consumer |
| :--- |
| samples the new |
| product and updates |
| his/her belief. | | Stage 3: The monopolistic |
| :--- |
| company sets the price $p$ |
| for the new product. |

Figure 4. The Timing of Game

Stage 1: The monopolistic seller sets the product sampling level $s$.
The company can use product sampling to provide the consumer with an opportunity to reduce his/her uncertainty and understand how much he/she values this new product before making the final purchase decision.

Stage 2: The consumer samples the new product and updates his/her belief.
The consumer has a prior belief for this new product before the product sampling $s$. After the product sampling $s$, the consumer will update his/her belief with an unfavourable/favourable impression.

Stage 3: The monopolistic seller sets the price $p$ for the new product.
After the product sampling $s$, the company can set the price for the new product according to the consumer posterior belief for this new product.

Stage 4: The consumer decides whether or not to buy the new product.
If the price $p$ for the new product is smaller than the consumer's belief for this product, the consumer will buy it. However, if the price $p$ is larger than the consumer's belief, the consumer will not buy it.

### 3.2 Consumer Prior Belief

At the outset (before product sampling), both the monopolistic seller and consumer do not know the consumer's valuation of the new product. However, they share a common belief that the consumer's valuation of the new product follows the distribution $F(v)$ :

$$
F(v)=\left\{\begin{array}{lll}
V L, & \text { with } & \text { probability } \\
V H, & \text { with } & \text { probability } \\
V
\end{array}\right.
$$

This is the consumer prior belief distribution for this new product, which can be high with probability of $\theta \in(0,1)$ and the value equals to $V H$ or be low with the probability of $1-\theta$ and the value equals to $V L$. The consumer with the low prior belief $V L$ is a poor fit for this new product and is not interested in buying it. The consumer with the high prior belief $V H$ is a good fit for this new product and is interested in buying it. The proportion of the low prior belief consumer is $\theta$, and The proportion of the low prior belief consumer is $1-\theta$.

### 3.3 Consumer Initial Uncertainty

According to the prior literature, consumers have initial uncertainty about product functionality, product quality, etc. This uncertainty can make people less likely to try it, which is seen by some as a reason why a product may not succeed in the market. To address this issue, companies often offer free trials to help potential customers feel more confident about trying their product. This approach benefits the companies by making it more likely that people will ultimately buy the product, which is why free trials are so common nowadays (Cheng and Liu 2012).

### 3.3.1 Signal Device

In this paper, we address the consumer's initial uncertainty to be $1-a$. The figure below shows how consumer initial uncertainty affects between the state of world and signal realisation.


Figure 5. Consumer Initial Uncertainty as A Signal Device

Because of consumer initial uncertainty, the consumer may have a signal device, which means the consumer will have a favourable/unfavourable impression on this new product. It is a mapping from the state of the world to the signal realisation. During the sampling of the product, the consumer with an unfavourable impression believes that the new product does not meet his/her needs. The consumer with a favourable impression believes that the new product is more likely to meet his/her needs, but the consumer may not be entirely sure that this is the case.

### 3.4 Product Sampling

In our study, we highlight the effect of product sampling in reducing uncertainty. The company can use product sampling to provide consumers with an opportunity to reduce their uncertainty and understand how much they value this new product before making the final purchase decision. The monopolistic company in different industries can choose to use which
and how many product sampling strategies and product sampling features to reduce the consumer's uncertainty. This is a choice variable for the monopolistic company, which is named as product sampling level $s$ in this paper.

### 3.4.1 Signal Device

During the product sampling, the consumer may also have a signal device. Figure 6 shows how product sampling level $s$ affects between the state of world and signal realisation based on Figure 5. In terms of the consumer with a low prior belief on this new product, a higher level of product sampling increases the likelihood that this consumer will get an unfavourable impression. And on the other hand, a higher level of product sampling increases the likelihood that a consumer who has a high prior belief for the new product will get a favourable impression. This may be explained by the fact that the consumer will have a clearer sense of his/her true value for this new product with a higher level of product sampling.

Unobserved Observed
The State of World
Product Sampling s


Figure 6. Product Sampling as A Signal Device

In the following of this paper, we will introduce three game theory models in order to solve the research question. Based on the three game theory models developed, a comprehensive analysis strategy can be outlined to help companies make informed decisions when introducing new products to the market.

The benchmark model establishes a basic understanding of the optimal product pricing and profit margins for a monopolistic company in the absence of consumer valuation
heterogeneity and initial uncertainty. This provides a foundation for comparison with the subsequent models that introduce additional factors such as consumer valuation heterogeneity and initial uncertainty. The high type consumer extension model extends the benchmark model to include the impact of consumer valuation heterogeneity on product pricing and profit. This model highlights the importance of considering the varying valuations that consumers may have for a product, and how this can impact the optimal pricing and profit margins for a company. The technical effect model builds on the high type consumer extension model by introducing the effect of technology improvement on reducing consumer uncertainty.

By utilising these models, companies can gain a better understanding of the complex dynamics that influence consumer behaviour and make data-driven decisions when introducing new products to the market.

## 4. Benchmark Model

### 4.1 Consumer Posterior Belief

We first built our benchmark model, which refers to the paper from Boleslavsky (2016). In this model, we assume that after product sampling, only the high-type consumer will generate an unfavourable/favourable impression. After the product sampling, the monopolistic company can take a survey and collect the feedback from the consumer. According to the feedback, the company can know whether or not the consumer has a favourable/unfavourable impression. Therefore, both the monopolistic company and the consumer can observe the signal realisation (favourable/unfavourable impression). The monopolistic company and the consumer share the same posterior belief after the product sampling $s$. After the product sampling, the consumer uncertainty will decrease by the product sampling level $s$. Therefore, the probability of having unfavourable impression is $s+a$, and the probability of having favourable impression is $1-(s+a)$.

Therefore, based on the product sampling level $s$ and the consumer initial uncertainty $1-a$, the distribution of consumer posterior belief for this new product $F^{\prime}(v \mid s)$ should be:

$$
F^{\prime}(v \mid s)=\left\{\begin{array} { c c c c } 
{ V L , } & { \text { with } } & { \text { probability } } & { 1 - \theta } \\
{ V H , } & { \text { with } } & { \text { probability } } & { \theta }
\end{array} \left\{\begin{array}{clll}
\text { unfavorable } & \text { impression, } & \text { with probability } & s+a \\
\text { favorable } & \text { impression, } & \text { with } & \text { probability }
\end{array} 1-(s+a)\right.\right.
$$

According to this consumer posterior belief, the probability of the consumer with a favourable impression is

$$
\operatorname{Pr}(\text { favourable impression }), p f=(1-(s+a))(1-\theta)+\theta
$$

The probability of the consumer with an unfavourable impression is

$$
\operatorname{Pr}(u n f a v o u r a b l e ~ i m p r e s s i o n), p u=(1-\theta)(s+a)
$$

We can calculate the conditional probability:

$$
\begin{aligned}
& \operatorname{Pr}(v=V L \mid \text { favourable impression })=(1-(s+a))(1-\theta) \\
& /[(1-(s+a))(1-\theta)+\theta] \\
& \operatorname{Pr}(v=V H \mid \text { favourable impression })=\theta /[(1-(s+a))(1-\theta)+\theta] \\
& \operatorname{Pr}(v=V L \mid \text { unfavourable impression })=1
\end{aligned}
$$

Therefore, the expected value for the belief of the consumer with favourable impression $E[v \mid$ favourable impression $], v f$ should be:

$$
\begin{aligned}
E[v \mid \text { favourable impression }]= & V L * \operatorname{Pr}(v=V L \mid \text { favourable impression }) \\
& +V H * \operatorname{Pr}(v=V H \mid \text { favourable impression }) \\
= & {[V L(1-\theta)(1-(s+a))+V H \theta] /[(1-(s+a))(1-\theta)+\theta] }
\end{aligned}
$$

On the other hand, the expected value for the belief of consumer with unfavourable impression $E[v \mid$ unfavourable impression $], v u$ should be:

$$
\begin{aligned}
E[v \mid \text { unfavourable impression }] & =V L * \operatorname{Pr}(v=V L \mid \text { unfavourable impression }) \\
+ & V H * \operatorname{Pr}(v=V H \mid \text { unfavourable impression }) \\
& =V L
\end{aligned}
$$

Therefore, the distribution of the expectation value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$ should be:

$$
\Gamma=\left\{\begin{array}{ccccc}
{[V L(1-\theta)(1-(s+a))+V H \theta] /[(1-\theta)(1-(s+a))+\theta],} & \text { with } & \text { probability } & (1-\theta)(1-(s+a))+\theta, & \text { favorable impression } \\
V L, & \text { with } & \text { probability } & (1-\theta)(s+a), & \text { unfavorable impression }
\end{array}\right.
$$

### 4.2 Solution

To simplify the model, we firstly set $s+a$ to be $m$ and solve the model.
According to the distribution of the expectation value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$, the monopolistic company can set the price of the new product.

Because $v f \geq v u$,

$$
\begin{gathered}
V L(1-\theta)(1-m)+V H \theta /[(1-m)(1-\theta)+\theta] \geq V L \\
V L(1-\theta)(1-m)+V H \theta \geq V L[(1-m)(1-\theta)+\theta] \\
V L \theta \geq V L \theta
\end{gathered}
$$

No conflict.
Therefore, $v f \geq v u$.

The monopolistic company can set the price to be $v f$ or $v u$.
(1) Set $p=V L(1-\theta)(1-m)+V H \theta$

If the monopolistic company set the price of the new product to be $v f$, then $p=v f$ and $p \geq v u$. Therefore, the consumer with a favourable impression will buy this new product because the price equals his/her expected value. On the other hand, the consumer with an unfavourable impression will not buy this new product because the price is larger than his/her expected value.

The demand $d$ should be: $d=p f=(1-m)(1-\theta)+\theta$
Therefore, the profit for the monopolistic company is:

$$
\pi 1(p)=d^{*} p=V L(1-\theta)(1-m)+V H \theta
$$

(2) Set $p=v u=V L$

If the monopolistic company set the price of the new product to be $v u$, then $p \leq v f$ and $p=v u$. Therefore, both the consumer with a favourable impression and an unfavourable impression will buy this new product because the price is smaller than or equals to his/her expected value.

The demand $d$ should be: $d=1$
Therefore, the profit for the monopolistic company is:

$$
\pi 2(p)=d^{*} p=V L
$$

After we get the profits for the monopolistic company, we need to compare $\pi 1(p)$ and $\pi 2(p)$ to find which one is larger. Then the monopolistic company should set the price $p$ according to the one which has a larger profit.

To compare $\pi 1(p)$ and $\pi 2(p)$, I assume $\pi 1(p) \geq \pi 2(p) \Rightarrow \pi 1(p) / \pi 2(p) \geq 1$

$$
\begin{gathered}
\pi 1(p) / \pi 2(p)=(1-\theta)(1-m)+\theta V H / V L \\
=1-\theta-m+\theta m+\theta V H / V L \\
1-\theta-m+\theta m+\theta V H / V L \geq 1 \\
\theta V H / V L \geq \theta+m-\theta m \\
V H / V L \geq 1+m / \theta-m
\end{gathered}
$$

Therefore, $\pi 1(p)>\pi 2(p)$ when $V H / V L \geq 1-m+m / \theta$. The monopolistic company should set the price of the new product $p$ to be $v f=V L(1-\theta)(1-m)+V H \theta$. Then the profit $\pi(p)=\pi 1(p)=V L(1-\theta)(1-m)+V H \theta$.

To get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi(p)$.

$$
d \pi(p) / d s<0
$$

$d \pi(p) / d s<0$, which means $\pi(p)$ is a monotonic decreasing function. Therefore, when $s^{*}=0$ the monopolistic company can have the maximum profit. The optimal price for the new product $p^{*}$ is $V L(1-\theta)+V H \theta$, and the maximum profit $\pi(p) *$ is

$$
V L(1-\theta)+V H \theta
$$

To sum up, $\pi 1(p)<\pi 2(p)$ when $V H / V L \leq 1-m+m / \theta$. The monopolistic company should set the price of the new product $p$ * to be $v u=V L$. Then the profit $\pi(p)^{*}=\pi 2(p)=V L$.

### 4.3 Welfare Analysis

In order to do the welfare analysis, we need to first calculate consumer surplus (cs) and total welfare (tw). Consumer surplus is the difference between the maximum price a consumer is willing to pay for a product and the actual price they pay for it. Total welfare, on the other hand, is the sum of the consumer surplus and the producer surplus. It represents the total economic benefit of a transaction to society.

1. When $V H / V L \geq 1-(s+a)+(s+a) / \theta$,

$$
\begin{aligned}
& \pi(p)^{*}=(1-\theta) V L+\theta V H \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& \quad=\theta(1-\theta)(V H-V L) \\
& t w=\pi+c s=\left(\theta^{2}-2 \theta+1\right) V L+\left(\theta-\theta^{2}+1\right) V H
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $>0$ | $*$ | $*$ |

Because $d \pi(p)^{*} / d \theta>0$, when $\theta$ increases, $\pi(p)^{*}$ will also increase.
2. When $V H / V L \leq 1-(s+a)+(s+a) / \theta$,

$$
\begin{aligned}
& \pi(p)^{*}=V L \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& \quad=\theta(V H-V L) \\
& t w=\pi+c s=(1-\theta) V L+\theta V H
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| 0 | $>0$ | $>0$ |

$d \pi(p)^{*} / d \theta=0$, when $\theta$ increases, $\pi(p) *$ will keep constant.
$d c s / d \theta>0$, when $\theta$ increases, $c s$ will increase.
$d t w / d \theta>0$, when $\theta$ increases, $t w$ will increase.

### 4.4 Conclusion

According to the solution above, the following tables conclude the optimal prices and optimal profits under two situations: 1. High consumer valuation heterogeneity; 2. Low consumer valuation heterogeneity. This consumer valuation heterogeneity represents $V H / V L$ in the model.

|  |  | $p^{*}$ | $\pi^{*}$ | $s^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| Consumer <br> Valuation <br> Heterogeneity | Low | High | $(1-\theta) \mathrm{VL}+\theta \mathrm{VH}$ | $(1-\theta) \mathrm{VL}+\theta \mathrm{VH}$ |

Figure 7. Optimal Profit, Price, Product Sampling Level Table for Model 1

When consumer valuation heterogeneity is high, which means the ratio of the valuation for the high-type consumer and the valuation of the low-type consumer is high, ${ }^{1}$ The high-type consumers are willing to pay more because they consider these goods of high value. However, the low-type consumers may think these products are unnecessary and are not willing to pay a high price.

When consumer valuation heterogeneity is low, which means the ratio of the valuation for the high-type consumer and the valuation of the low-type consumer is low, ${ }^{2}$ The high-type consumers and low-type consumers have similar valuation for these products.

According to Figure 7, when the consumer valuation heterogeneity is low, the optimal product sampling level is 0 , and when the consumer valuation heterogeneity is high, all the product sampling levels will have the same optimal price and profit.

Under the condition of low consumer valuation heterogeneity, product sampling hurts the optimal profit, and under the condition of high consumer valuation heterogeneity, the optimal profit does not depend on product sampling. Therefore, we created a second model based on this benchmark model in order to find meaningful product sampling level strategies.

[^0]
## 5. Extension for High-type Consumer

### 5.1 Consumer Posterior Belief

Based on the benchmark model, we modified that after product sampling, both the high-type consumer and low-type consumer may generate an unfavourable/favourable impression. Similarly, after the product sampling, the monopolistic company can take a survey and collect the feedback from the consumer. According to the feedback, the company can know whether or not the consumer has a favourable/unfavourable impression. The monopolistic company and consumer share the same posterior belief after the product sampling $s$. The probability of low-type consumer having unfavourable impression is $s+a$, and the probability of low-type consumer having favourable impression is $1-(s+a)$. The probability of high-type consumer having favourable impression is $s+a$, and the probability of high-type consumer having unfavourable impression is $1-(s+a)$.

Therefore, based on the product sampling level $s$ and the consumer initial uncertainty $1-a$, the distribution of consumer posterior belief for this new product $F^{\prime}(v \mid s)$ should be:

According to this consumer posterior belief, the probability of the consumer with a favourable impression is:

$$
\operatorname{Pr}(\text { favourable impression }), p f=(1-(s+a))(1-\theta)+\theta(s+a)
$$

The probability of the consumer with an unfavourable impression is:

$$
\operatorname{Pr}(\text { unfavourable impression }), p u=(1-\theta)(s+a)+\theta(1-(s+a))
$$

We can calculate the conditional probability:
$\operatorname{Pr}(v=V L \mid$ favourable impression $)=(1-(s+a))(1-\theta) /[(1-(s+a))(1-\theta)+\theta(s+a)$
$\operatorname{Pr}(v=V H \mid$ favourable impression $)=\theta(s+a) /[(1-(s+a))(1-\theta)+\theta(s+a)]$
$\operatorname{Pr}(v=V L \mid$ unfavourable impression $)=(1-\theta)(s+a) /[(1-\theta)(s+a)+\theta(1-(s+a))]$
$\operatorname{Pr}(v=V H \mid$ unfavourable impression $)=\theta(1-(s+a)) /[(1-\theta)(s+a)+\theta(1-(s+a))]$

Therefore, the expected value for the belief of the consumer with favourable impression $E[v \mid$ favourable impression $], v f$ should be:
$E[v \mid$ favourable impression $]=V L * \operatorname{Pr}(v=V L \mid$ favourable impression $)$

$$
+V H * \operatorname{Pr}(v=V H \mid \text { favourable impression })
$$

$$
=[V L(1-(s+a))(1-\theta)+V H \theta(s+a)] /[(1-(s+a))(1-\theta)+\theta(s+a)]
$$

On the other hand, the expected value for the belief of consumer with unfavourable impression E[v|unfavourable impression], vu should be:

$$
\begin{aligned}
E[v \mid \text { unfavourable impression }] & =V L^{*} \operatorname{Pr}(v=V L \mid \text { unfavourable impression }) \\
& +V H^{*} \operatorname{Pr}(v=V H \mid \text { unfavourable impression })
\end{aligned}
$$

Therefore, the distribution of the expectation value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$ should be:

$$
\Gamma=\left\{\begin{array}{lllll}
{[V L(1-\theta)(1-(s+a))+V H \theta(s+a)] /[(1-(s+a))(1-\theta)+\theta(s+a)],} & \text { with } & \text { probability } & (1-\theta)(1-(s+a))+\theta(s+a), & \text { favorable } \\
\text { impression } \\
{[V L(1-\theta)(s+a)+V H \theta(1-(s+a))] /[(1-\theta)(s+a)+\theta(1-(s+a))],} & \text { with } & \text { probability } & (1-\theta)(s+a)+\theta(1-(s+a)), \quad \text { unfavorable } & \text { impression }
\end{array}\right.
$$

The solution process is similar to the one in 4.2 , we will ignore it here. Please find more details in the Appendix.

### 5.2 Welfare Analysis

1. When $V H / V L<1 / \theta+1$,

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

$$
\begin{aligned}
& \pi(p)^{*}=(1-\theta) V L+\theta V H \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& =\theta(1-\theta)(V H-V L) \\
& t w=\pi+c s=\left(\theta^{2}-2 \theta+1\right) V L+\left(\theta-\theta^{2}+1\right) V H
\end{aligned}
$$

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $>0$ | $*$ | $*$ |

Because $d \pi(p)^{*} / d \theta>0$, when $\theta$ increases, $\pi(p)^{*}$ will also increase.
2. When $V H / V L \geq 1 / \theta+1 \& 0<a<1 / 2$,

$$
\begin{aligned}
& \pi(p)^{*}=(1-\theta) a V L+\theta(1-a) V H \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& =[\theta(1-\theta) a(V H-V L)] /[(1-\theta) a+\theta(1-a)] \\
& t w=\pi+c s= \\
& \quad[\theta(1-\theta) a(V H-V L)] /[(1-\theta) a+\theta(1-a)] \\
& \quad \quad+(1-\theta) a V L+\theta(1-a) V H
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $<0$ | $*$ | $*$ |

$d \pi(p) * / d \theta<0$, when $\theta$ increases, $\pi(p) *$ will decrease.

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $a$, we can get this table.

| $d \pi(p)^{*} / d a$ | $d c s / d a$ | $d t w / d a$ |
| :---: | :---: | :---: |
| $<0$ | $*$ | $*$ |

$d \pi(p) * / d a<0$, when $a$ increases, $\pi(p) *$ will decrease.
3. When $V H / V L \geq 1 / \theta+1 \& 1 / 2<a<1$,
$\pi(p)^{*}=V H \theta$
$c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta$

$$
\begin{aligned}
& =0 \\
t w & =\pi+c s=V H \theta
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $>0$ | 0 | $>0$ |

$d \pi(p)^{*} / d \theta>0$, when $\theta$ increases, $\pi(p) *$ will increase.
$d c s / d \theta=0$, when $\theta$ increases, $c s$ will keep constant.
$d t w / d \theta>0$, when $\theta$ increases, $t w$ will increase.

### 5.3 Conclusion

According to the solution before, the following tables summarise the optimal prices and optimal profits under three situations: 1 . consumer valuation heterogeneity is low ; 2 . consumer valuation heterogeneity is high \& consumer initial uncertainty is high. 3. consumer valuation heterogeneity is high and consumer initial uncertainty is low. This consumer valuation heterogeneity represents $V H / V L$ in the model.


Figure 10. Optimal Profit Table for Model 2


Figure 11. Optimal Price Table for Model 2

|  |  | Consumer initial uncertainty |  |
| :---: | :---: | :---: | :---: |
|  | High | Low |  |
| Consumer <br> Valuation <br> Heterogeneity | Low | 1/2-a |  |

Figure 12. Optimal Product Sampling Level Table for Model 2

According to Figure 13, when consumer valuation heterogeneity is low, as consumer initial uncertainty increases from 0 to 0.5 , the optimal product sampling level $\mathrm{s}^{*}$ is 0 . When consumer initial uncertainty increasing from 0.5 to 1 , the optimal product sampling level s* increases from 0 to 0.5 . When consumer valuation heterogeneity is high, as consumer initial uncertainty increases from 0 to 0.5 , the optimal product sampling level s* increases from 0 to 0.5 . When consumer initial uncertainty increasing from 0.5 to 1 , the optimal product sampling level s* is 0 .


Figure 13. Optimal Product Sampling Level Plot

To explain this result in the real world, we created a use-case table for these three conditions, which provides some examples for each condition in the real world. As shown in Figure 14, basic digital products and daily necessities are under the condition: consumer valuation heterogeneity is low. For basic digital products and daily necessities, both high-type consumers and low-type consumers have clearer recognition for their valuation. Therefore, the consumer valuation heterogeneity is low.

For high-tech products, high-type consumers may pay more, however, low-type consumers may not be willing to spend too much on these because high-tech products are not necessary for them. Therefore, high-tech products have high consumer valuation heterogeneity. On the other hand, for high-tech products, there were no analog products in the market before. Therefore, the consumer initial uncertainty for high-tech products is high.

For luxury, similar to high-type products, the ratio of valuation for high-type consumer and low-type consumer is high. Therefore, luxury has high consumer valuation heterogeneity. On the other hand, for luxury, there were analog products in the market before. Therefore, the consumer initial uncertainty for luxury is low.


Figure 14. Use Case in Real World

Combining the examples in Figure 14 in order to explain the results in real world, we can get:

For basic digital products or daily necessities, the company should not produce product sampling if the consumer initial uncertainty is low. The optimal product sampling level should increase from 0 to 0.5 when the consumer uncertainty is high (increases from 0.5 to 1).

For high-tech products, the company should not produce product sampling in order to get the optimal profit.

For luxury, the optimal product sampling level should increase from 0 to 0.5 when the consumer uncertainty increases from 0 to 0.5 .

## 6. Technical Effect

Technical effect refers to the impact of technological advancements on a product's performance and its effects on consumer behaviour. In the context of the technical effect model, the model introduces the technical effect factor to capture the influence of improved technology on the reduction of consumer uncertainty. The model considers the changes in consumer beliefs and behaviour after the product sampling, taking into account both the initial uncertainty and the technical effect. By incorporating the technical effect into the model, we can better understand the effects of technological advancements on consumer behaviour and decision-making, and how companies can optimise their pricing and product strategies to maximise profits. The technical effect model provides insights into the optimal pricing, product sampling level, and profit levels for different types of products based on consumer valuation heterogeneity and initial uncertainty.

### 6.1 Consumer Posterior Belief

Based on model 2, we introduced technical effect $\lambda$ and built model 3 .

This technical effect $\lambda$ represents the influence on the reduction of consumer uncertainty after technology improvement. With the improvement of technology, consumer uncertainty will reduce more than before. Therefore, $\lambda \geq 1$.

The monopolistic company and consumer share the same posterior belief after the product sampling $s$. After introducing technical effect $\lambda$, the probability of low-type consumer having unfavourable impression is $\lambda(s+a)$, and the probability of low-type consumer having favourable impression is $1-\lambda(s+a)$. The probability of high-type consumer having favourable impression is $\lambda(s+a)$, and the probability of high-type consumer having unfavourable impression is $1-\lambda(s+a)$.

Therefore, based on the product sampling level $s$, the consumer initial uncertainty $1-a$, and technical effect $\lambda$, the distribution of consumer posterior belief for this new product $F^{\prime}(v \mid s)$ should be:

According to this consumer posterior belief, the probability of the consumer with a favourable impression

$$
\operatorname{Pr}(\text { favorable impression }), p f=(1-\lambda(s+a))(1-\theta)+\theta \lambda(s+a)
$$

The probability of the consumer with an unfavourable impression

$$
\operatorname{Pr}(\text { unfavourable impression }), p u=(1-\theta) \lambda(s+a)+\theta(1-\lambda(s+a))
$$

We can calculate the conditional probability:

Therefore, the expectation value for the belief of the consumer with favourable impression $E[v \mid$ favourable impression $]$, vf should be:

$$
\begin{aligned}
E[v \mid \text { favourable impression }]= & V L * \operatorname{Pr}(v=V L \mid \text { favourable impression }) \\
& +V H * \operatorname{Pr}(v=V H \mid \text { favourable impression })
\end{aligned}
$$

$$
=[V L(1-\lambda(s+a))(1-\theta)+V H \theta \lambda(s+a)] /[(1-\lambda(s+a))(1-\theta)+\theta \lambda(s+a)]
$$

On the other hand, the expectation value for the belief of consumer with unfavourable impression $E[v \mid u n f a v o u r a b l e ~ i m p r e s s i o n], ~ v u ~ s h o u l d ~ b e: ~: ~$
$E[v \mid$ unfavourable impression $]=V L^{*} \operatorname{Pr}(v=V L \mid$ unfavourable impression $)$ $+V H * \operatorname{Pr}(v=V H \mid u n f a v o u r a b l e ~ i m p r e s s i o n) ~$
$=[V L(1-\theta) \lambda(s+a)+\operatorname{VH\theta }(1-\lambda(s+a))] /[(1-\theta) \lambda(s+a)+\theta(1-\lambda(s+a))]$

$$
\begin{aligned}
& \operatorname{Pr}(v=V L \mid \text { favorable impression })=(1-\lambda(s+a))(1-\theta) /[(1-\lambda(s+a))(1-\theta) \\
& +\theta \lambda(s+a)] \\
& \operatorname{Pr}(v=V H \mid \text { favorable impression })=\theta \lambda(s+a) /[(1-\lambda(s+a))(1-\theta)+\theta \lambda(s+a)] \\
& \operatorname{Pr}(v=V L \mid u n f a v o u r a b l e ~ i m p r e s s i o n)=(1-\theta) \lambda(s+a) /[(1-\theta) \lambda(s+a) \\
& +\theta(1-\lambda(s+a))] \\
& \operatorname{Pr}(v=V H \mid u n f a v o u r a b l e ~ i m p r e s s i o n ~) ~=\theta(1-\lambda(s+a)) /[(1-\theta) \lambda(s+a) \\
& +\theta(1-\lambda(s+a))]
\end{aligned}
$$

Therefore, the distribution of the expectation value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$ should be:
$\Gamma=\left\{\begin{array}{lllll}{[V L(1-\theta)(1-\lambda(s+a))+V H \theta \lambda(s+a)] /[(1-\lambda(s+a))(1-\theta)+\theta \lambda(s+a)],} & \text { with } & \text { probability } & (1-\theta)(1-\lambda(s+a))+\theta \lambda(s+a), \quad \text { favorable } \quad \text { impression } \\ {[V L(1-\theta) \lambda(s+a)+V H \theta(1-\lambda(s+a))] /[(1-\theta) \lambda(s+a)+\theta(1-\lambda(s+a))],} & \text { with } & \text { probability } & (1-\theta) \lambda(s+a)+\theta(1-\lambda(s+a)), \quad \text { unfavorable } & \text { impression }\end{array}\right.$

The solution process is similar to the one in 4.2 , we will ignore it here. Please find more details in the Appendix.

### 6.2 Welfare Analysis

1. When $V H / V L<1 / \theta+1$,

$$
\begin{aligned}
& \pi(p)^{*}=(1-\theta) V L+\theta V H \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& \quad=\theta(1-\theta)(V H-V L) \\
& t w=\pi+c s=\left(\theta^{2}-2 \theta+1\right) V L+\left(\theta-\theta^{2}+1\right) V H
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $>0$ | $*$ | $*$ |

Because $d \pi(p)^{*} / d \theta>0$, when $\theta$ increases, $\pi(p)^{*}$ will also increase.
2. When $V H / V L \geq 1 / \theta+1 \& 0<a<1 / 2$,

$$
\begin{aligned}
& \pi(p)^{*}=(1-\theta) \lambda a V L+\theta(1-\lambda a) V H \\
& c s=\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& =[\theta(1-\theta) \lambda a(V H-V L)] /[(1-\theta) \lambda a+\theta(1-\lambda a)] \\
& t w=\pi+c s=[\theta(1-\theta) \lambda a(V H-V L)] /[(1-\theta) \lambda a+\theta(1-\lambda a)] \\
& \quad \quad+(1-\theta) \lambda a V L+\theta(1-\lambda a) V H
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $<0$ | $*$ | $*$ |

$d \pi(p) * / d \theta<0$, when $\theta$ increases, $\pi(p) *$ will decrease.

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $a$, we can get this table.

| $d \pi(p)^{*} / d a$ | $d c s / d a$ | $d t w / d a$ |
| :---: | :---: | :---: |
| $<0$ | $*$ | $*$ |

$d \pi(p)^{*} / d a<0$, when $a$ increases, $\pi(p)^{*}$ will decrease.
3. When $V H / V L \geq 1 / \theta+1 \& 1 / 2<a<1$,

$$
\begin{aligned}
& \pi(p)^{*}=V H \theta \\
& \begin{aligned}
c s & =\left(V L-p^{*}\right) \times(1-\theta)+\left(V H-p^{*}\right) \times \theta \\
& =0
\end{aligned} \\
& t w=\pi+c s=V H \theta
\end{aligned}
$$

By calculating the derivatives of $\pi(p)^{*}$, cs, tw based on $\theta$, we can get this table.

| $d \pi(p)^{*} / d \theta$ | $d c s / d \theta$ | $d t w / d \theta$ |
| :---: | :---: | :---: |
| $>0$ | 0 | $>0$ |

$d \pi(p)^{*} / d \theta>0$, when $\theta$ increases, $\pi(p)^{*}$ will increase.
$d c s / d \theta=0$, when $\theta$ increases, $c s$ will keep constant.
$d t w / d \theta>0$, when $\theta$ increases, $t w$ will increase.

### 6.3 Conclusion

According to the solution before, the following tables conclude the optimal prices and optimal profits under three situations: 1 . consumer valuation heterogeneity is low ; 2 . consumer valuation heterogeneity is high \& consumer initial uncertainty is high. 3. consumer valuation heterogeneity is high \& consumer initial uncertainty is low. This consumer valuation heterogeneity represents $V H / V L$ in the model.


Figure 15. Optimal Profit Table for Model 3

Optimal Price Table - 3


Figure 16. Optimal Price Table for Model 3


Figure 17. Optimal Product Sampling Level Table for Model 3

Combined with the use case table in Figure 18, our conclusion in the real world should be:

For basic digital products or daily necessities, the company should not produce product sampling if the consumer initial uncertainty is low. The optimal product sampling level should increase from 0 to $1 / 2 \lambda$ when the consumer uncertainty is high (from 0.5 to 1 ). The optimal sampling level $s^{*}$ is smaller than that of model 2 under the same condition.

For high-tech products, the company should not produce product sampling in order to get the optimal profit. The optimal price and profit are smaller than those of model 2.

For luxury products, the optimal product sampling level should increase from 0 to $1 / 2 \lambda$ when the consumer uncertainty increases from 0 to 0.5 . The optimal sampling level $s *$ is smaller than that of model 2 under the same condition.

|  |  | Consumer initial uncertainty |  |
| :---: | :---: | :---: | :---: |
|  | High | Low |  |
| Consumer <br> Valuation <br> Heterogeneity | Low | Basic Digital / Daily | Necessities |

Figure 18. Use Case in Real World

## 7. Limitation

In terms of the limitation for this paper, there are several points that can be improved and concluded in the future study.

Firstly, in this paper we only considered the situation where the monopolistic company implemented product sampling first, and then set the price for the new product. However, there is one more condition for the timeline. The monopolistic company can also set the price first and then carry out the product sampling strategies. These two situations of timeline may cause differences for the final result. We can compare and conclude which condition is better for the company according to these differences and help the monopolistic company make a better strategy for the product sampling.

Secondly, in terms of the favourable and unfavourable impression generated by the consumers. We assume that both high-type consumer and low-type consumer have the same probability to generate the correct impression respective to their valuation. However, in the real world, the probability for the high-type consumer to generate a favourable impression and the probability for the low-type consumer to generate an unfavourable impression may not be the same. We can build another model considering this situation and make the conclusion according to the result in the future study.

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## Appendix

## Model Solution for High-type Extension Model

To simplify the model, we also set $s+a$ to be $m$ and solve the model.
According to the distribution of the expected value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$, the monopolistic company can set the price of the new product. This may have two conditions: 1.vf $\geq v u ; 2 . v f \leq v u$

1. When $v f \geq v u$,

$$
\begin{gathered}
V L(1-\theta)(1-m)+V H \theta m /[(1-m)(1-\theta)+\theta m] \geq \\
V L(1-\theta)+V H \theta(1-m) /(1-\theta) m+\theta(1-m) \\
V L(1-\theta)(1-2 m) \geq V H(1-\theta)(1-2 m) \\
\because V H \geq V L
\end{gathered}
$$

Therefore, when $v f \geq v u, m \geq 1 / 2$.

The monopolistic company can set the price to be $v f$ or $v u$.
(1) $\operatorname{Set} p=v f$

If the monopolistic company set the price of the new product to be $v f$, then $p=v f$ and $p \geq v u$. Therefore, the consumer with a favourable impression will buy this new product because the price equals his/her expected value. On the other hand, the consumer with an unfavourable impression will not buy this new product because the price is larger than his/her expected value.
The demand $d$ should be: $d=p f=(1-(s+a))(1-\theta)+\theta(s+a)$
Therefore, the profit for the monopolistic company

$$
\pi 1(p)=d^{*} p=V L(1-\theta)(1-m)+V H \theta m
$$

(2) Set $p=v u$

If the monopolistic company set the price of the new product to be $v u$, then $p \leq v f$ and $p=v u$. Therefore, both the consumer with a favourable impression and an unfavourable impression will buy this new product because the price is smaller than or equals to his/her expected value.

The demand $d$ should be: $d=1$
Therefore, the profit for the monopolistic company

$$
\begin{gathered}
\pi 2(p)=d^{*} p=V L(1-\theta)(s+a)+V H \theta(1-(s+a)) /(1-\theta)(s+a) \\
+\theta(1-(s+a))
\end{gathered}
$$

After we get the profits for the monopolistic company, we need to compare $\pi 1(p)$ and $\pi 2(p)$ to find which one is larger. Then the monopolistic company should set the price $p$ according to the one which has a larger profit.

To compare $\pi 1(p)$ and $\pi 2(p)$, I assume $\pi 1(p) \geq \pi 2(p) \Rightarrow \pi 1(p) / \pi 2(p) \geq 1$

$$
\begin{gathered}
{[V L(1-m)(1-\theta)+V H \theta m] /[V L(1-\theta) m+V H \theta(1-m)] \geq} \\
1 /[(1-\theta) m+\theta(1-m)] \\
(1-m)(1-\theta) V L+V H \theta m \geq V L \\
(-m-\theta+m \theta) V L+V H \theta m \geq 0 \\
V H / V L \geq(m+\theta-m \theta) / m \theta \\
V H / V L \geq 1 / \theta+1
\end{gathered}
$$

Therefore, $\pi 1(p)>\pi 2(p)$ when $V H / V L \geq 1 / \theta+1$.

When $V H / V L \geq 1 / \theta+1$, the monopolistic company should set the price of the new product $p$ to be $v f$. Then the profit is $\pi 1(p)=V L(1-(s+a))(1-\theta)+V H \theta(s+a)$ When $V H / V L \leq 1 / \theta+1$, the monopolistic company should set the price of the new product $p$ to be $v u$. Then the profit is

$$
\pi 2(p)=[V L(1-\theta)(s+a)+V H \theta(1-(s+a))] /[(1-\theta)(s+a)+\theta(1-(s+a))]
$$

When $V H / V L \geq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 1(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 1(p)$.

$$
d \pi 1(p) / d s>0
$$

$d \pi 1(p) / d s>0$, which means $\pi 1(p)$ is a monotone increasing function. Therefore, when $m^{*}=1, s^{*}=1-a$, the monopolistic company can maximise its profit. The optimal price for the new product $p^{*}$ is $V H$, and the maximum profit $\pi(p) *$ is $V H \theta$.

When $V H / V L \geq 1 / \theta+1$. The monopolistic company should set the price of the new product $p$ * to be $V H$. Then the profit $\pi(p)^{*}=V H \theta$.

When $V H / V L \leq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 2(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 2(p)$.

$$
d \pi 2(p) / d s<0
$$

$d \pi 2(p) / d s<0$, which means $\pi(p)$ is a monotone decreasing function. Therefore, when $m^{*}=1 / 2, s^{*}=1 / 2-a$, the monopolistic company can have the maximum profit. The optimal price for the new product $p$ *is $V L(1-\theta)+V H \theta$, and the maximum profit $\pi(p) *$ is $V L(1-\theta)+V H \theta$.

When $V H / V L \leq 1 / \theta+1$. The monopolistic company should set the price of the new product $p^{*}$ to be $V L(1-\theta)+V H \theta$. Then the profit $\pi(p)^{*}=V L(1-\theta)+V H \theta$.
2. When $v f \leq v u$,

$$
\begin{gathered}
V L(1-\theta)(1-m)+V H \theta m /[(1-m)(1-\theta)+\theta m] \leq \\
V L(1-\theta)+V H \theta(1-m) /(1-\theta) m+\theta(1-m) \\
V L(1-\theta)(1-2 m) \leq V H(1-\theta)(1-2 m) \\
\because V H \geq V L
\end{gathered}
$$

Therefore, when $v f \leq v u, m \leq 1 / 2$.

The monopolistic company can set the price to be $v f$ or $v u$.
(1) Set $p=v f$

If the monopolistic company set the price of the new product to be $v f$, then $p=v f$ and $p \leq v u$. Therefore, the consumer with a favourable impression will buy this new product because the price equals his/her expected value. On the other hand, the consumer with an unfavourable impression will also buy this new product because the price is smaller than his/her expected value.
The demand $d$ should be: $d=1$
Therefore, the profit for the monopolistic company

$$
\pi 1(p)=d^{*} p=[V L(1-m)(1-\theta)+V H \theta m] /[(1-m)(1-\theta)+\theta m]
$$

(3) Set $p=v u$

If the monopolistic company set the price of the new product to be $v u$, then $p \geq v f$ and $p=v u$. Therefore, the consumer with an unfavourable impression will not buy this new product because the price is larger than or equals to his/her expectation value.
The demand $d$ should be: $d=p u$
Therefore, the profit for the monopolistic company

$$
\pi 2(p)=d^{*} p=V L(1-\theta) m+V H \theta(1-m)
$$

After we get the profits for the monopolistic company, we need to compare $\pi 1(p)$ and $\pi 2(p)$ to find which one is larger. Then the monopolistic company should set the price $p$ according to the one which has a larger profit.

To compare $\pi 1(p)$ and $\pi 2(p)$, I assume $\pi 1(p) \leq \pi 2(p) \Rightarrow$

$$
\begin{gathered}
{[V L(1-\theta) m+V H \theta(1-m)] /[V L(1-m)(1-\theta)+V H \theta m] \geq} \\
1 /[(1-m)(1-\theta)+\theta m] \\
(1-\theta) m+V H \theta(1-m) / V L \geq 1 \\
V H(\theta-\theta m) \geq V L(1+m \theta-m) \\
V H / V L \geq(1+m \theta-m) / \theta(1-m) \\
V H / V L \geq 1 / \theta+1
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$$

Therefore, $\pi 1(p) \leq \pi 2(p)$ when $V H / V L \geq 1 / \theta+1$.

When $V H / V L \geq 1 / \theta+1$, the monopolistic company should set the price of the new product $p$ to be $v u$. Then the profit $\pi 1(p)=V L(1-\theta)(s+a)+V H \theta(1-(s+a))$ When $V H / V L \leq 1 / \theta+1$, the monopolistic company should set the price of the new product $p$ to be $v f$. Then the profit
$\pi 2(p)=[V L(1-\theta)(1-(s+a))+V H \theta(s+a)] /[(1-\theta)(1-(s+a))+\theta(s+a)]$

When $V H / V L \geq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 1(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 1(p)$.

$$
d \pi 1(p) / d s<0
$$

$d \pi 1(p) / d s<0$, which means $\pi 1(p)$ is a monotone decreasing function. Therefore, when $m^{*}=a, s^{*}=0$, the monopolistic company can have the maximum profit. The optimal price for the new product $p$ *is
$[V L(1-\theta) a+V H \theta(1-a)] /[(1-\theta) a+\theta(1-a)]$, and the maximum profit $\pi(p)$ * is $V L(1-\theta) a+V H \theta(1-a)$.

When $V H / V L \geq 1 / \theta+1$. The monopolistic company should set the price of the new product $p$ * to be $[V L(1-\theta) a+V H \theta(1-a)] /[(1-\theta) a+\theta(1-a)]$. Then the profit $\pi(p)^{*}=V L(1-\theta) a+V H \theta(1-a)$.

When $V H / V L \leq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 2(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 2(p)$.

$$
d \pi 2(p) / d s>0
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$d \pi 2(p) / d s>0$, which means $\pi(p)$ is a monotone increasing function. Therefore, when $m^{*}=1 / 2, s^{*}=1 / 2-a$, the monopolistic company can have the maximum profit. The optimal price for the new product $p^{*}$ is $V L(1-\theta)+V H \theta$, and the maximum profit $\pi(p) *$ is $V L(1-\theta)+V H \theta$.

When $V H / V L \leq 1 / \theta+1$. The monopolistic company should set the price of the new product $p^{*}$ to be $V L(1-\theta)+V H \theta$. Then the profit $\pi(p)^{*}=V L(1-\theta)+V H \theta$.

## Model Solution for Technical Effect Model

To simplify the model, we also set $\lambda(s+a)$ to be $m$ and solve the model.
According to the distribution of the expectation value of the consumer posterior belief for consumer with favourable/unfavourable impression $\Gamma$, the monopolistic company can set the price of the new product. This may have two conditions: 1.vf $\geq v u ; 2 . v f \leq v u$

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\begin{gathered}
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The demand $d$ should be: $d=p f=(1-\lambda(s+a))(1-\theta)+\theta \lambda(s+a)$
Therefore, the profit for the monopolistic company

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\pi 1(p)=d^{*} p=V L(1-\theta)(1-m)+V H \theta m
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The demand $d$ should be: $d=1$
Therefore, the profit for the monopolistic company

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\begin{gathered}
\pi 2(p)=d^{*} p=V L(1-\theta) \lambda(s+a)+V H \theta(1-\lambda(s+a)) /(1-\theta) \lambda(s+a) \\
+\theta(1-\lambda(s+a))
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After we get the profits for the monopolistic company, we need to compare $\pi 1(p)$ and $\pi 2(p)$ to find which one is larger. Then the monopolistic company should set the price $p$ according to the one which has a larger profit.

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When $V H / V L \geq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 1(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 1(p)$.

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1 /[(1-m)(1-\theta)+\theta m] \\
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V H(\theta-\theta m) \geq V L(1+m \theta-m) \\
V H / V L \geq(1+m \theta-m) / \theta(1-m) \\
V H / V L \geq 1 / \theta+1
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Therefore, $\pi 1(p) \leq \pi 2(p)$ when $V H / V L \geq 1 / \theta+1$.

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$$
\pi 2(p)=[V L(1-\theta)(1-\lambda(s+a))+V H \theta \lambda(s+a)] /[(1-\theta)(1-\lambda(s+a))+\theta \lambda(s+a)]
$$

When $V H / V L \geq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 1(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 1(p)$.

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$[V L(1-\theta) \lambda a+V H \theta(1-\lambda a)] /[(1-\theta) \lambda a+\theta(1-\lambda a)]$, and the maximum profit $\pi(p){ }^{*}$ is $V L(1-\theta) \lambda a+V H \theta(1-\lambda a)$.

When $V H / V L \geq 1 / \theta+1$. The monopolistic company should set the price of the new product $p^{*}$ to be $[V L(1-\theta) \lambda a+V H \theta(1-\lambda a)] /[(1-\theta) \lambda a+\theta(1-\lambda a)]$. Then the profit $\pi(p)^{*}=V L(1-\theta) \lambda a+V H \theta(1-\lambda a)$.

When $V H / V L \leq 1 / \theta+1$, to get the optimal product sampling level $s^{*}$, we need to calculate the product sampling level $s$ when $s$ can maximise the profit $\pi 2(p)$. In order to get $s^{*}$, we calculate the derivative of $\pi 2(p)$.

$$
d \pi 2(p) / d s>0
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$d \pi 2(p) / d s>0$, which means $\pi(p)$ is a monotone increasing function. Therefore, when $m^{*}=1 / 2, s^{*}=1 / 2 \lambda-a$, the monopolistic company can have the maximum profit. The optimal price for the new product $p *$ is $V L(1-\theta)+V H \theta$, and the maximum profit $\pi(p) *$ is $V L(1-\theta)+V H \theta$.

When $V H / V L \leq 1 / \theta+1$. The monopolistic company should set the price of the new product $p^{*}$ to be $V L(1-\theta)+V H \theta$. Then the profit $\pi(p)^{*}=V L(1-\theta)+V H \theta$.


[^0]:    ${ }^{1}$ Products such as high-tech products, luxury, and art products have high consumer valuation heterogeneity.
    ${ }^{2}$ Products such as basic digitals and daily necessities have low consumer valuation heterogeneity.

